Effective Frequency Selection Algorithm for Bandpass Sampling of Multiband RF Signals Based on Relative Frequency Interval

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***Abstract*—In design of software receiver, costs of AD converter is greatly decreased and data amount is largely reduced by using bandpass sampling. However, the interval of sampled signal in spectrum is ignored in related researches, especially in multiband RF signals' sampling by single frequency. To solve the problem, we introduce RFI (Relative Frequency Interval) as one important criterion to evaluate performance of sampling frequency. Based on RFI, we propose and infer an effective selection algorithm to get the optimal sampling frequency. In application of GPS receiver, we implement the proposed algorithm with numerical method to get available sampling frequency, which directly determines data amount and the corresponding RFI value. Then according to actual demand, we can get the optimal sampling frequency.**

Keywords-bandpass sampling; multiband RF signal; sampling frequency; relative frequency interval (RFI)

I. INTRODUCTION

In software receiver design, AD converter should be placed as near the antenna as possible. So to omit the down-converter bank, bandpass sampling is used in AD converter. The sampling not only samples the signal but also shifts RF band to base band in spectrum.

For bandpass sampling, the former work mainly focuses on searching for the minimum sampling frequency to reduce the data amount to be processed after sampling. Satyabrata Sen[1] gave an algorithm to search for minimum frequency based on some constraints without considering the inverse placement of frequency band. Chi Chen[2] gave an additional correction of constraints and added the guard band factor. Subhonmesh[3], Dennis M. Akos[4], Ching-Hsiang[5] etc also did the similar work.

However, what they all ignored is the frequency spectrum after sampling, especially the interval between adjacent signals. Jianhua Liu[6] considered the problem of normal spectral placement and inverse spectral placement of sampled signals that sampling of single RF band may cause. He proposed constraints about selecting sampling frequency to satisfy the evenly spaced distribution of sampled signals. But there is still no evaluation and solution for multiband sampling.

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In this paper, the point we lay emphases on is the frequency interval of sampled RF signals, which directly determines the difficulty of following digital bandpass filter. The relative interval between sampled frequency bands is introduced to be a criterion to evaluate the sampling performance. Generally, the bigger the interval is, the better the sampling frequency is. Because the relative interval between sampled signals heavily influences the design of following digital filter. For this reason, we should consider about data amount and relative interval together to find the most optimum sampling frequency.

II. BANDPASS SAMPLING THEOREM

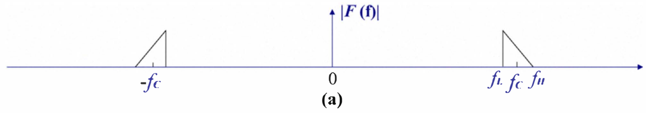

Figure 1. Spectrum of single signals

As Figure 1, a band-limited signal is defined by its center frequency f_c and band region (f_L, f_H) .Bandwidth of the signal is $B = f_H - f_L$ .According to Bandpass Sampling Theory, the sampling frequency of band-limited signal must meet the following constraints.

Normal spectral placement (As Figure 2):

$$\frac{2f_c+B}{2n+1} \le f_s \le \frac{2f_c-B}{2n} \quad \text{where} \quad 0 \le n \le [\frac{2f_c-B}{4B}] \quad (1)$$

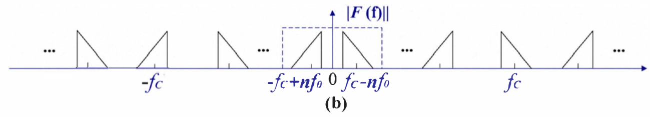

Figure 2. Sampled signal of normal spectral placement

Reverse spectral placement (As Figure 3):

$$\frac{2f_c+B}{2n} \le f_s \le \frac{2f_c-B}{2n-1} \quad \text{where} \quad 0 \le n \le [\frac{2f_c+B}{4B}] \quad (2)$$

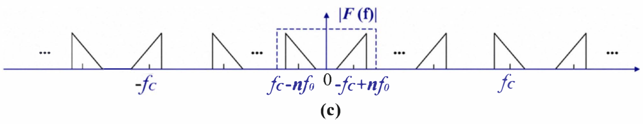

Figure 3. Sampled signal of inverse spectral placement

In the above equations, positive integer n is frequency shift coefficient. The integral function $[x]$ is the function that its value is the biggest integer less than the independent variable x or equal to it.

III. Frequency Selection in Bandpass Sampling of Multiband RF Signals

A. *Relative Frequency Interval (RFI)*

In reference [6], Jianhua Liu considered about the infinite bandwidth of transition zone in design of digital filters and tried to make the sampled signals evenly spaced spanning in spectrum, and got the constraints for single signal.

For sampling n signals at the same time, the definition of *Relative Frequency Interval* or *RFI* is introduced to evaluate the sampling performance.

Relative Frequency Interval: After frequency spectrum of multiband signal is shifted by sampling, the minimum ratio of frequency interval between center frequency of every two signals ($\left|f_i - f_j\right|$) to half the sum of their bandwidth ($\frac{1}{2}\left|B_i + B_j\right|$). RFI's mathematical expression is as Equation (13).

$$RFI = \min\left\{\frac{|f_i - f_j|}{(B_i + B_j)/2}, \frac{|f_i + f_j - f_s|}{(B_i + B_j)/2}\right\} \tag{3}$$

It is clear that the condition of anti-aliasing is $\mathrm{RFI} \geq 1$, under which the frequency f_s is a desired one. Moreover, the greater RFI is, the larger interval exists between sampled signals. As a result, larger frequency interval contributes to broader transition zone, which makes the design of digital filters less difficult.

B. *Theoretical Analysis of Mutliband Bandpass Sampling*

Assume there are n narrow-band RF signals. Now analyze how to choose the sampling frequency f_s to assure no sampled frequency band aliasing in the 1st positive Nyquist Band $[0, \frac{f_s}{2}]$ and the 1st negative Nyquist Band $[-\frac{f_s}{2}, 0]$. Meanwhile, we will keep an eye on increasing intervals between adjacent bands in order to decrease design difficulty of digital filters.

Firstly, according to spectrum periodicity of sampled signals, we can equally deal with signals in 1st positive Nyquist Band ($[0, \frac{f_s}{2}]$) instead of the whole spectrum. The non-aliasing constraints can be described as following:

- After sampling, the interval between center frequencies of every two sampled signals should be equal to or greater than half the bandwidth of the two RF signals.
- Due to the symmetry of real signal's spectrum, there exists the constraint that spectrum of sampled signal should not include point $n \cdot \frac{f_s}{2}$.

Let $f_{c1}, f_{c2}, \cdots, f_{cN}$ be the center frequency of the N narrow-band RF signals separately in frequency spectrum and $B_1, B_2, \cdots, B_N$ be their bandwidth separately.

The equal mathematical expression of above two constraints can be represented as Equation (4), (5) and (6).

$$\left|\left(f_{ci} - \left[\frac{f_{ci}}{f_s}\right]\cdot f_s\right) - \left(f_{cj} - \left[\frac{f_{cj}}{f_s}\right]\cdot f_s\right)\right| \geq \frac{B_i + B_j}{2} \tag{4}$$

$$\left|\left(-f_{ci} + \left[\frac{f_{ci}}{f_s}\right]\cdot f_s + f_s\right) - \left(f_{cj} - \left[\frac{f_{cj}}{f_s}\right]\cdot f_s\right)\right| \geq \frac{B_i + B_j}{2} \tag{5}$$

$$f_{ci} - \left[\frac{f_{ci}}{f_{cs}}\right]\cdot f_s \geq \frac{B_i}{2} \tag{6}$$

where $i, j = 1, 2, \cdots, n$ and $i \neq j$

Using the symbol rem(x, y) as remainder operator, which means $rem(x, y) = x - [\frac{x}{y}] \cdot y$, the above equations can be simplified as Equation (7), (8), (9).

$$\left|rem(f_{ci}, f_s) - rem(f_{cj}, f_s)\right| \geq \frac{B_i + B_j}{2} \tag{7}$$

$$\left|rem(f_{ci}, f_s) + rem(f_{cj}, f_s) - f_s\right| \geq \frac{B_i + B_j}{2} \tag{8}$$

$$rem(f_{ci}, f_s) \geq \frac{B_i}{2} \tag{9}$$

where $i, j = 1, 2, \cdots, n$ and $i \neq j$.

Secondly, according to Nyquist-Shannon Sampling Theorem[7] [8], the sampling frequency must be no less than twice of sum of all narrow-band signals' bandwidth. The corresponding expression is as Equation (10).

$$f_s \geq 2 \cdot \sum_{i=1}^{n} B_i \tag{10}$$

Thirdly, sampling frequency should be smaller than the sampling frequency in Low-pass Sampling Theorem generally. It can be described as Equation (10).

$$f_s \le \max_i \{f_{ci} + \frac{B_i}{2}\} \qquad (11)$$

In practice, to deal with an inverse spectral placement in the 1st Nyquist Band, we need to spend some efforts on reversing the high and low frequency components. To avoid that, it's better to shift normal instead of inverse spectral placement into 1st Nyquist band. The constraints of normal spectral placement are as Equation (11).

For the ith narrow band signal, set $f_i = rem(f_{ci}, f_s)$, then

$$f_i \le \frac{f_s}{2}, \text{ where } i = 1, 2, \cdots, n \qquad (12)$$

Similarly, to achieve inverse spectral placement, the following equation should be meet.

$$f_i \ge \frac{f_s}{2}, \text{ where } i = 1, 2, \cdots, n \qquad (13)$$

Accordingly, we successfully convert the problem about how to choose the sampling frequency in given multiband RF signals to a mathematical question about how to find proper frequency f_s that satisfies Equation (7), (8), (9), (10), (11), (12)(or(13)).

C. Flow Chart of Effective Sampling Frequency Selection Algorithm

Even though we abstract the mathematical expressions from engineering problems successfully. It is still very difficult to solve the equations set (include (7), (8), (9), (10), (11), (12) (or (13)) through analytical approach. Also, it's not worth to get the analytical solution. Therefore, we propose a numerical solution based on exhaustive method to find the right frequency satisfying all above equations.

The flow chart of the numerical solution is as Figure 4.

As Figure 4, according to the equations set (include (7), (8), (9), (10), (11) and (12) (or (13)), we design the algorithm to minimize the calculated amount. By implementing the algorithm with Matlab, we can get corresponding RFI value attached to every sampling frequency. The step of circulating can be set by users depending on the hardware performance.

In practice, after we get available sampling frequency, we need to make some choice. The higher sampling frequency is, the larger data amount of the sampled signals is. Meanwhile, the larger RFI is, the lower order of digital filter is. So we should comprehensively combine the influence of sampling frequency and RFI to find the most proper sampling frequency.

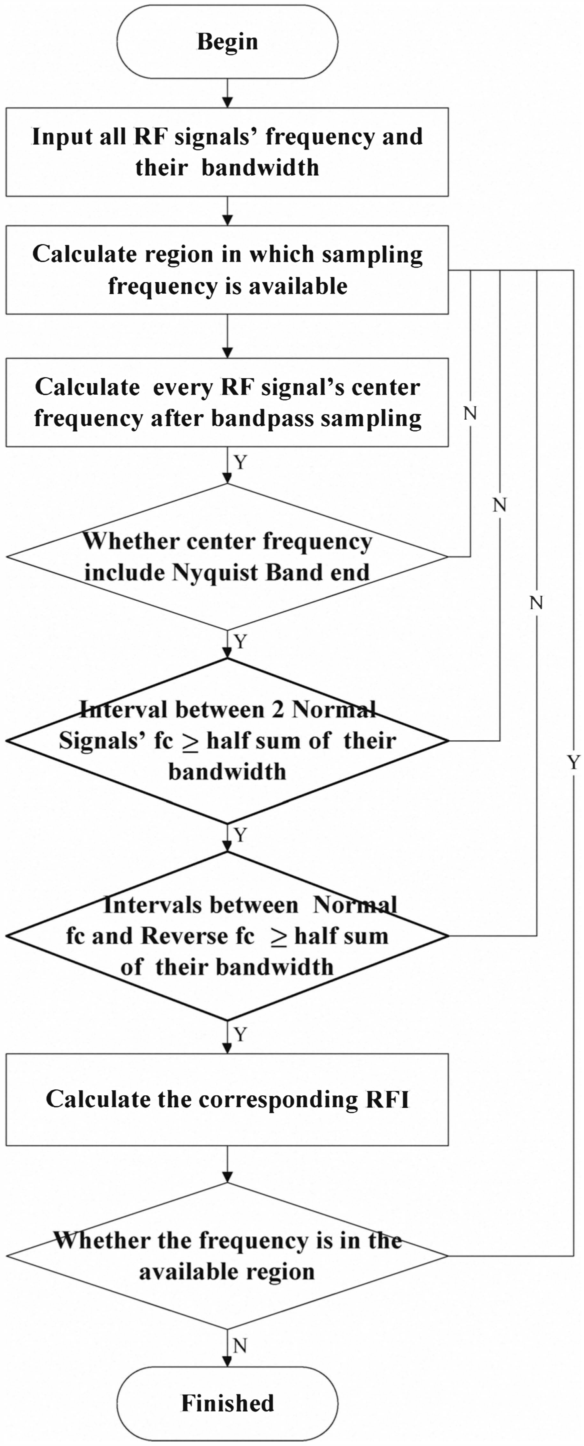

Figure 4. Flow Chart of Effective Sampling Frequency Selection Algorithm

IV. PERFORMANCE IN PRACTICE

A. Direct RF Sampling in GPS Receiver

GPS signals' features are shown in Table 1.

TABLE I. GPS SIGNAL PARAMETERS

Signal Identification (ID)	Center Frequency (MHz)	Bandwidth (MHz)
L1(C/A)	1575.42	2
L2(C)	1227.60	20
L5	1176.45	20

The design of GPS receiver inevitably involves the processing of multiband signals. Generally, the existing solution is pre-filtering and down-converting respectively for each frequency band to get the desired signal. But this always leads to poor coherence of signals, which may causes omission in the following tracing and capturing process. What's more, it needs many down-converters and filters bank which will increases system costs greatly.

To overcome above mentioned difficulties, we can sample GPS signal directly at RF band based on the multiband bandpass sampling theory. Direct RF sampling significantly reduces costs and complexity of SDR system and enhance the coherence of signals.

B. Sampling Frequency Selection of Multiband GPS Receiver

1) By implementing the algorithm in Figure 4, we can get the RFI value of available frequency for bandpass sampling L1 and L2 band, which is shown in Figure 5 and Figure 6.

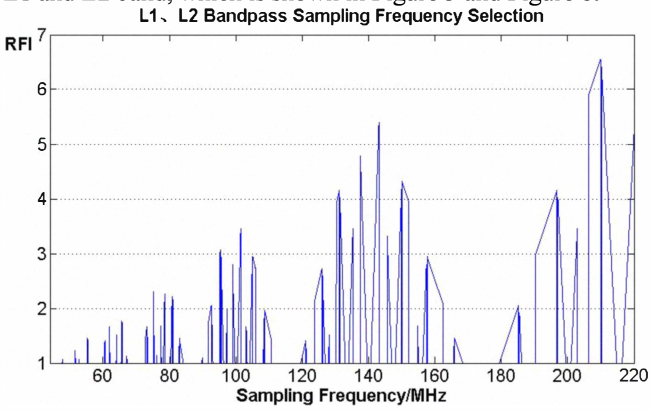

Figure 5. L1, L2 Bandpass Sampling Frequency Selection

From figure 5, it can be concluded that the minimum sampling frequency is not the best choice because its RFI is just about 1, which means the digital filter transition zone is almost zero. It is very hard to design a qualified filter since very little transition zone requires very high order filter.

Besides, we can find that choosing 168MHz is not a good choice, either. Even though 168MHz meets the constraints of bandpass sampling, its low RFI makes it much difficult to design following digital filter.

Taking both RFI and data amount into consideration, 100MHz is a very good choice. The value of RFI is about 3, which assures that shifted spectrum leaves enough transition zone to digital filter. And the data amount is not very large at the same time.

2) As the number of bands increases, frequency spectrum is becoming more and more crowded. The result of sampling L1, L2 and L5 is shown in Figure 6.

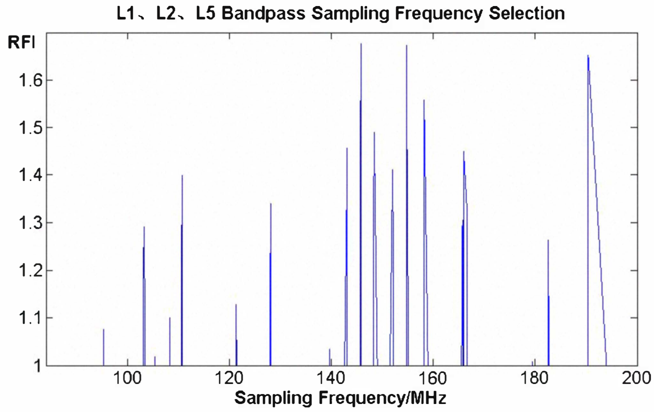

Figure 6. L1, L2, L5 Bandpass Sampling Frequency Selection

In Figure 6, the minimum sampling frequency 95MHz's RFI is less than 1.1, which means there is very narrow transition zone in spectrum. However, 110MHz's RFI increases to 1.4, which is much more than that of 95MHz. Meanwhile, we find that RFI of the frequency around 120MHz is almost as low as 95MHz's. It illustrates that higher sampling frequency is not the necessary condition of higher RFI.

In reference[9], for the same question, the author gave a solution at 222MHz and an alternative solution of 158MHz. However, from Figure 6, we find RFI reaches the acme at about 146 MHz. So from the aspect of frequency interval, 146MHz instead of 158MHz is the best choice.

V. CONCLUSION

Former research on multiband bandpass sampling mainly focuses on searching for the minimum sampling frequency without taking sampled spectrum distribution into account. Thus highly crowded spectrum will make the design of digital filters rather difficult.

Aiming at this problem, in this paper, first we introduce a new definition of RFI to evaluate the degree of spectrum crowdedness after sampling. Then based on RFI, we propose and infer an effective frequency selection algorithm for bandpass sampling of multiband RF signals. Through the algorithm, we can not only get available sampling frequency, but also calculate the RFI value under the corresponding sampling frequency to estimate its performance. Finally, we implement the algorithm with numerical method in multiband GPS signal bandpass sampling and figure out how to choose the optimal sampling frequency under different actual demands.

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